A Discrete Time Pricing Model for Individual Insurance Contracts

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Abstract: Most ratemaking principles or models for insurance pricing in the literature do not seem to incorporate at least one of the following three elements: contingency of claims, investment income of insurers, and insolvency risk of insurers. As far as we know, Doherty-Garven (1986) appears to be the only model that incorporates all the three components. Their model provides a solution for the "fair" rate of return of an insurer or the *aggregate premium* of an insurer. Their study has some important implications in terms of "excessiveness" and "adequacy" of the *aggregate premium*. In this paper, we developed a pricing model with which a premium can be assigned to an insurance contract. Our effort may have some important implications in terms of the "fairness" of the *individual premium* of an insurance contract.

INTRODUCTION

A gross premium includes a net premium and an expense load. The net premium is for claim payments. Meanwhile, the expense load is for expenses such as agent commissions, operating expenses, taxes, and so forth. We focus on only the net premium, and the expense load is not considered in this paper. Therefore, hereafter, an insurance premium is regarded as the value of the payoff to an insurance contract.

Regulators require that an insurance premium should not be excessive, inadequate, or unfairly discriminatory.[1] The excessiveness and the adequacy are related to the aggregate premium of the insurance portfolio held by an insurer. Meanwhile, the fairness is associated with the individual

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Journal of Insurance Issues, 2004, **27**, 1, pp. 41–65.

premium of an insurance contract. An insurer first estimates an insurance contract and assigns an individual premium based upon the measurement and the ratemaking principle chosen by the insurer or a regulator. Therefore, the critical issues associated with the excessiveness, adequacy, and fairness of an insurance premium are the measurement of the insurance contract and the choice set of available ratemaking principles.

Although most ratemaking principles in the actuarial literature consider the contingency of claims, they tend to ignore the investment income of insurers and the insolvency risk of insurers. However, the investment income of insurers should be an important part of insurance pricing because of the time lag between premium receipts and claim payments. Furthermore, the insolvency risk of insurers is not negligible.[2]

Unlike the ratemaking principles in the actuarial literature, the previous financial models for insurance pricing consider the investment income of insurers as an important determinant of an insurance premium in addition to the contingency of claims. Most of them, however, do not seem to incorporate the insolvency risk of insurers except Doherty and Garven (1986), whose model incorporates the insolvency risk of an insurer and provides a solution for the fair rate of return of an insurer or the *aggregate premium* of the insurance portfolio held by an insurer. Nevertheless, an *individual premium* cannot be assigned to an insurance contract by using their model.

The primary objective of our research is to derive a model with which an *individual premium* could be assigned to an insurance contract, satisfying the following requirements

- The contingency of claims, the insolvency risk of insurers, and the investment income of insurers are incorporated according to valid theoretical justifications.
- An *individual premium* is equal to the equilibrium price of the payoff to an insurance contract.
- The *aggregate premium* is equal to the equilibrium price of the payoff to the insurance portfolio held by an insurer.

The main focus or motivation of our study is different from that of Doherty and Garven (1986). Our study focuses on the fairness of individual premiums, whereas their study addresses the excessiveness and adequacy of aggregate premiums.

LITERATURE REVIEW

The literature associated with our research can be classified into two groups. The first group includes the ratemaking principles in the actuarial literature, and the financial models for insurance pricing make up the second group.

Among the ratemaking principles[3] in the actuarial literature, the principles using moments are the most naive ones. As a measure of contingency of claims, they employ various moments, such as expected value, standard deviation, variance, and semi-variance. Meanwhile, there are other ratemaking principles employing various statistical parameters other than moments. The representative ones are the maximal loss principle and probability of ruin principle. Most sophisticated ratemaking principles in the actuarial literature are functional principles, such as the mean value principle and zero utility principle. They introduce risk premiums for the contingency of claims by employing some functions. Although the ratemaking principles in the actuarial literature consider the contingency of claims, the employed risk measures and risk prices are not based on strong theoretical justifications. Furthermore, most of them ignore the investment income of insurers and the insolvency risk of insurers.

Since Ferrari (1968), many financial models for insurance pricing have been presented.[3-1] A key notion of the previous models is that an insurance premium should reflect an equilibrium relationship between risk and return in a competitive market. To derive models for insurance pricing, various financial models, such as the Capital Asset Pricing Model (CAPM), the Arbitrage Pricing Theory (APT), the Discounted Cash Flow Model (DCFM), the Option Pricing Model (OPM), and the Discrete Time Model for Contingent Claims (DTMCC), are applied.

Cooper (1974), Bigger and Kahane (1978), Fairley (1979), Hill (1979), and Hill and Modigliani (1987) applied the CAPM to develop a model for the appropriate underwriting profit margin of an insurer. Their approaches are based on the equilibrium relationship between the insurance market and the capital market. Doherty and Kang (1988) is another example of the application of the CAPM in the context of the underwriting cycle. Although the insurance CAPMs[4] incorporate the investment income of insurers and the contingency of claims, they tend to ignore the insolvency risk of insurers.

Kraus and Ross (1982) assumed that the claims from the insured follow a process governed by a differential equation, and the economic factors follow a diffusion process. They applied APT and Ito's lemma to derive a model for insurance pricing. Like the insurance CAPMs, their model incorporates the investment income of insurers and the contingency of claims, but does not consider the insolvency risk of insurers.

The representative models for insurance pricing based on the Discounted Cash Flow Method are Myers and Cohn (1987) and the National Council on Compensation Insurance Model, that employs the net present value method and internal rate of return method, respectively. Consequently, the essential part of the models is determining the risk-adjusted discount rate or cost of capital. Although these models incorporate the investment income of insurers and the contingency of claims, they do not consider the insolvency risk of insurers.

After Black and Scholes (1973) argued that the equity of a leveraged firm could be considered as a call option on the value of the firm, there have been many studies applying OPM for insurance pricing. Among the insurance pricing models based on OPM, Smith (1979) is a frontier study, but has some problems. First, the value of an insured asset generally does not follow a geometric Brownian motion. Second, the claim from an insurance contract is not equal to the difference between the insured asset value and the insured value. Third, the insolvency risk of insurers is ignored.

To overcome the first problem, Merton (1978), Pennacchi (1987), and Cummins (1988) introduced the discrete jumps in the process of the insured asset value. Because of the second problem, some studies applied OPM to only few specific cases. For example, Merton (1977) and Ronn and Verma (1986) applied OPM to the deposit insurance, whereas Cummins (1988) applied it to insurance guaranty funds. Doherty and Garven (1986) went one step further and presented a model for insurance pricing that incorporates the insolvency risk of insurers in addition to the investment income of insurers and the contingency of claims. They applied the CAPM, Rubinstein (1976), and the DTMCC to derive the fair rate of return of an insurer.

On the other hand, D'Arcy and Gorvett (1998) provided a comprehensive comparison among financial models for insurance pricing. As the first step of the comparison, a fictitious but representative insurer was generated. The authors then selected seven financial models for insurance pricing and applied each model to the fictitious insurer to calculate appropriate underwriting profit margins. They compared the underwriting profit margins of the models and analyzed the strength and weakness of each model. Their study provides a guideline for understanding how financial models for insurance pricing could be applied to real-world situations.

Most financial models for insurance pricing have focused on the underwriting profit margin of an insurer or the aggregate premium of the insurance portfolio held by an insurer. Phillips, Cummins, and Allen (1998) provided a good method of disaggregating the aggregate premium by lines. Their dynamic model and empirical results show that the insurance price is negatively related to a firm's risk, especially for long-tail lines. Also, they found that the line-specific growth rates have a significant effect on insurance pricing. Although they developed a model that disaggregates the aggregate premium by lines, but they did not address individual premiums. In this paper, we will attempt to disaggregate the aggregate premium into individual levels.

MODEL

As the first step of our model building, a simple insurer is postulated to identify the payoff to an insurance contract.

- The insurer starts its business at time $t = 0$ and ends its business at time $t = 1$.
- The insurer establishes its initial equity (S_0) at time $t = 0$.
- The insurer constructs an insurance portfolio at time $t = 0$.
- The insurer receives an aggregate premium (Π_a) from the insurance portfolio at time $t = 0$, where Π_a is the sum of individual premiums (Π_i) of insurance contracts in the portfolio.
- With the fund ($S_0 + \Pi_a$), the insurer constructs an investment portfolio at time $t = 0$.
- The insurer first pays an *aggregate claim* ($\tilde{L}_a$) from the insurance portfolio at time $t = 1$, where $\tilde{L}_a$ is the sum of *individual claims* ($\tilde{L}_i$) from the insurance contracts in the portfolio, and the payment priorities among individual contracts are the same.
- If there is a residual fund, the insurer pays the fund to its shareholders and ends its business at time $t = 1$.

Now we can identify the payoff to policyholders. If the insurer is not bankrupt, policyholders obviously are entitled to receive a legitimate claim payment ($\tilde{L}_i$). If the insurer is bankrupt, however, the terminal value of the insurance company $\{(S_0 + \Pi_a)(1 + \tilde{r}_A)\}$ will be distributed to policyholders according to the amount of each individual policyholder's claim. For example, if a policyholder's claim represents 1 percent of total claims, the policyholder receives 1 percent of the insurer's terminal value at bankruptcy. Therefore,

$$\textit{Payoff to an insurance contract} = Min\left\{L_i, (S_0 + \Pi_a)\left((1 + \tilde{r}_A)\frac{L_i}{L_a}\right)\right\}, \quad (1)$$

where $\tilde{r}_A$ = rate of return of the investment portfolio.

Since an individual premium (Π_i) is equal to the value of the payoff to an insurance contract,

$$\begin{aligned} \Pi_i &= V\left[Min\left\{L_i, (S_0 + \Pi_a)(1 + \tilde{r}_A)\frac{L_i}{L_a}\right\}\right] \quad (2) \\ &= V\left[L_i - Max\left\{L_i - (S_0 + \Pi_a)\frac{L_i}{L_a}, 0\right\}\right] \\ &= V[L_i] - V\left[Max\left\{L_i - (S_0 + \Pi_a)(1 + \tilde{r}_A)\frac{L_i}{L_a}, 0\right\}\right] \end{aligned}$$

where V[•] = value of •.

Please note that the first term of equation (2) is the value of individual claims, and the second term of the equation is the insolvency risk premium. Therefore equation (2) represents that an individual insurance premium is equal to the value of individual claims minus the insolvency risk premium. In other words, if an insurance company is at a higher risk of insolvency, the firm should offer a lower insurance premium than other firms at a lower risk of insolvency. It is obvious that we should pay a smaller amount of premium to the company that is subject to a higher probability of ruin.

This study employs two sets of pricing models and assumptions to solve equation (2) and eventually to derive a model for individual insurance contracts.

Sets	*Pricing Models*	*Assumptions*
First Set	* CAPM * Discrete Time Models for Contingent Claims	* Joint Normal Distribution * Constant Absolute Risk Aversion (CARA)
Second Set	* Rubinstein Model (1976) * Discrete Time Models for Contingent Claims	* Joint Lognormal Distribution * Constant Relative Risk Aversion (CRRA)

The normality, lognormality, CARA, and CRRA assumptions are frequently employed in the finance and insurance literature. For example, the mean-variance criterion has been employed to handle the topics associated with portfolio, and normality allows the coincidence between the mean-variance criterion and the expected utility criterion. Black and Scholes (1973) derived their option pricing model by assuming lognormality. Although there have been many studies on the shape of utility functions, the most frequently employed utility functions are exponential and power utility functions, which exhibit CARA and CRRA, respectively.

Joint Normality and Constant Absolute Risk Aversion (CARA) Case

To solve equation (2), we make the following assumptions.

- $\tilde{L}_i$, $\tilde{r}_A$, $\tilde{L}_a$, and the aggregate wealth are joint normally distributed.
- A representative investor exhibits CARA, meaning that the degree of risk aversion does not change regardless of wealth change.[5]

We apply the CAPM to solve the first term of equation (2). The result is that

$$V[\tilde{L}_i] = \frac{\hat{E}(\tilde{L}_i)}{1 + r_f}, \tag{3}$$

where $\hat{E}(\bullet)$ = certainty equivalent expectation of $\bullet$,

$$\hat{E}(\tilde{L}_i) = E(\tilde{L}_i) - \{E(\tilde{r}_m) - r_f\}\frac{Cov(\tilde{L}_i, \tilde{r}_m)}{\sigma_m^2},$$

r_f = risk-free interest rate,
E($\bullet$) = expectation of $\bullet$,
$\tilde{r}_m$ = market rate of return,
$Cov(\bullet)$ = covariance of $\bullet$,
σ_m^2 = variance of $\tilde{r}_m$.

Equation (3) states that the value of individual claims is a present value of certainty equivalent claims, which is the expected claim minus the risk premium for the contingency of claims. Therefore, if the underwriting beta of an individual claim is zero, then the insurance premium is equal to the present value of the expected claim discounted by a risk-free rate. Equation (3) implies that the higher the contingency of claims, the lower the insurance premium should be.

The second term of equation (2) is the same as the value of a call option whose underlying variable is $\tilde{L}_i - (S_0 + \Pi_a)(1 + \tilde{r}_A)\frac{\tilde{L}_i}{\tilde{L}_a}$ and whose exercise price is 0. The Stapleton and Subrahmanyam model (1984)[6] is applied to solve the second term of equation (2) with the synthesized variables: $\tilde{X}_1 = \tilde{L}_i$, $\tilde{X}_2 = (S_0 + \Pi_a)(1 + \tilde{r}_A)$, and $\tilde{X}_3 = \tilde{L}_a$.
The result is that

$$V\left[Max\left\{\tilde{L}_i - (S_0 + \Pi_a)(1 + \tilde{r}_A)\frac{\tilde{L}_i}{\tilde{L}_a}, 0\right\}\right] = V\left[Max\left\{\tilde{X}_1 - \frac{\tilde{X}_2\tilde{X}_1}{\tilde{X}_3}, 0\right\}\right] \tag{4}$$

$$= \frac{1}{1 + r_f}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\frac{1}{(2\pi)^{1.5}|\Sigma|^{.5}}Max\left\{X_1 - \frac{X_2X_1}{X_3}, 0\right\}$$

$$\bullet \quad exp\left\{-\frac{1}{2}\underline{X}'\Sigma^{-1}\underline{X}\right\}dX_1dX_2dX_3$$

where Σ = the variance and covariance matrix of $\tilde{X}_1, \tilde{X}_2, \tilde{X}_3$ $= \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}$

$$\sigma_{11} = Var(\tilde{X}_1) = \sigma^2_{\tilde{L}_i}$$

$$\sigma_{12} = \sigma_{21} = Cov(\tilde{X}_1, \tilde{X}_2) = (S_0 + \Pi_a)Cov(\tilde{L}_i, \tilde{r}_A)$$

$$\sigma_{13} = \sigma_{31} = Cov(\tilde{X}_1, \tilde{X}_3) = Cov(\tilde{L}_i, \tilde{L}_a)$$

$$\sigma_{22} = Var(\tilde{X}_2) = (S_0 + \Pi_a)^2\sigma^2_{\tilde{r}_A}$$

$$\sigma_{23} = \sigma_{32} = Cov(\tilde{X}_2, \tilde{X}_3) = (S_0 + \Pi_a)Cov(\tilde{L}_a, \tilde{r}_A)$$

$$\sigma_{33} = Var(\tilde{X}_3) = \sigma^2_{\tilde{L}_a}$$

$|\bullet|$ = determinant of $\bullet$

$$\underline{X} = \begin{bmatrix} X_1 - \hat{E}(\tilde{X}_1) \\ X_2 - \hat{E}(\tilde{X}_2) \\ X_3 - \hat{E}(\tilde{X}_3) \end{bmatrix}$$

$$\hat{E}(\tilde{X}_1) = E(\tilde{L}_i) - \{E(\tilde{r}_m) - r_f\} \frac{Cov(\tilde{L}_i, \tilde{r}_m)}{\sigma_m^2}$$

$$\hat{E}(\tilde{X}_2) = (S_0 + \Pi_a)(1 + r_f)$$

$$\hat{E}(\tilde{X}_3) = E(\tilde{L}_a) - \{E(\tilde{r}_m) - r_f\} \times \frac{Cov(\tilde{L}_a, \tilde{r}_m)}{\sigma_m^2}.$$

Please note that equation (4) simply represents the risk premium associated with an insurer's insolvency. Since an individual premium is equation (3) minus equation (4), the higher the insolvency risk of an insurance company, the lower the insurance premium should be.

Now we can determine an individual premium (Π_i) with equations (2), (3), and (4). The individual insurance premium (Π_i) is a function of Π_a, however, since the right hand side of equation (4) is a function of Π_a. Therefore, we need another model for the equilibrium aggregate premium (Π_a*) to obtain the equilibrium individual premium (Π_i*).

We consider the *payoff to the shareholders* of the postulated insurer to develop a model for Π_a*.[7] The *payoff to the shareholders* is either a residual fund (terminal value of an insurance company minus aggregate claims) or 0 for the case of insolvency. In other words,

$$\textit{Payoff to shareholders} = Max\{(S_0 + \Pi_a)(1 + \tilde{r}_A) - \tilde{L}_a, 0\}. \qquad (5)$$

Therefore,

$$V_e = [Max\{(S_0 + \Pi_a)(1 + \tilde{r}_A) - \tilde{L}_a, 0\}], \qquad (6)$$

where V_e = value of *the payoff to shareholders*.

The right-hand side of equation (6) is the same as the value of a call option whose underlying variable is $(S_0 + \Pi_a)(1 + \tilde{r}_A) - \tilde{L}_a$ and whose exercise price is 0. The Brennan model (1979)[8] is applied to solve equation (6) with a synthesized variable: $\tilde{X} = (S_0 + \Pi_a)(1 + \tilde{r}_A) - \tilde{L}_a$.

The result is that

$$V_e = \frac{1}{1 + \tilde{r}_f}\left\{\hat{E}(\tilde{X})N\left[\frac{\hat{E}(\tilde{X})}{\sigma_{\tilde{X}}}\right] + \sigma_{\tilde{X}}n\left[\frac{\hat{E}(\tilde{X})}{\sigma_{\tilde{X}}}\right]\right\}, \tag{7}$$

where $\hat{E}(\tilde{X}) = (S_0 + \Pi_a)(1 + \tilde{r}_f) - \hat{E}(\tilde{L}_a)$,

$$\hat{E}(\tilde{L}_a) = \hat{E}(\tilde{L}_a) - \{E(\tilde{r}_m) - r_f\}\frac{Cov(\tilde{L}_a, \tilde{r}_m)}{\sigma_m^2},$$

$\sigma_{\tilde{X}}^2$ = variance of $\tilde{X} = (S_0 + \Pi_a)^2\sigma_{\tilde{r}_A}^2 + \sigma_{\tilde{L}_a}^2 - 2(S_0 + \Pi_a)Cov(\tilde{L}_a, \tilde{r}_A)$,

N[ ] = c.d.f. of standard normal distribution, and
n[ ] = p.d.f. of standard normal distribution.[9]

Please note that the right-hand side of equation (7) is a function of Π_a, and V_e (the value of the payoff to shareholders) should be equal to S_0 (initial equity) in equilibrium. Therefore, Π_a* is the implicit solution of the following equation:

$$S_0 = \frac{1}{1 + r_f}\left\{\hat{E}(\tilde{X})N\left[\frac{\hat{E}(\tilde{X})}{\sigma_{\tilde{X}}}\right] + \sigma_{\tilde{X}}n\left[\frac{\hat{E}(\tilde{X})}{\sigma_{\tilde{X}}}\right]\right\}. \tag{8}$$

Equation (8) represents that the present value of the payoff to shareholders should be the same as the initial equity of the insurance company in equilibrium.

Equations (2), (3), (4), and (8) constitute a pricing model for an insurance contract. The procedures approaching Π_i* are as follows:

- Procedure 1: Solve equation (8) for Π_a* using an iterative procedure.
- Procedure 2: Solve equation (4) with the Π_a* obtained by Procedure 1.
- Procedure 3: Solve equation (3).

- Procedure 4: Solve equation (2) with the results obtained by Procedures 2 and 3.

Joint Lognormality and Constant Relative Risk Aversion (CRRA) Case

Here, we solve equation (2) under the following assumptions:

- $\tilde{L}_i$, $1 + \tilde{r}_f$, $\tilde{r}_A$, and aggregate wealth are joint lognormally distributed.
- A representative investor exhibits CRRA, meaning that the degree of risk aversion changes as wealth changes.

The procedures to derive a model in this section are the same as those of the *Joint Normality and CARA* case. The procedures involve solving the first and second terms of equation (2) and developing a model for $\Pi_a{}^*$.

The Rubinstein model (1976) is applied to solve the first term of equation (2). The result is that

$$V[\tilde{L}_i] = \frac{\hat{E}(\tilde{L}_i)}{1 + r_f}, \tag{9}$$

where $\hat{E}(\tilde{L}_i) = E(\tilde{L}_i) exp\{-\lambda Cov(1\text{n}\tilde{L}_i, 1\text{n}\tilde{R}_m)\}$,

λ = relative risk aversion parameter = $\dfrac{E(l\text{n}\tilde{R}_m) - 1\text{n}R_f}{\sigma^2_{1\text{n}\tilde{R}_m}} + \dfrac{1}{2}$,

$1\text{n}\,\bullet$ = The natural logarithm of $\bullet$,

$\tilde{R}_m = 1 + \tilde{r}_m$, and

$R_f = 1 + r_f$.

Please note that the meaning of equation (9) is exactly the same as that of equation (3).

The Stapleton and Subrahmanyam (1984)[10] model is applied to solve the second term of equation (2). The result is that

$$V\left[Max\left\{\tilde{L}_i - (S_0 + \Pi_a)(1 + \tilde{r}_A)\frac{\tilde{L}_i}{\tilde{L}_a}, 0\right\}\right] = V\left[Max\left\{\tilde{X}_1 - \left(\frac{\tilde{X}_2\tilde{X}_1}{\tilde{X}_3}, 0\right)\right\}\right] \tag{10}$$

$$= \frac{1}{1+r_f}\int_0^{+\infty}\int_0^{+\infty}\int_0^{+\infty}\frac{1}{(2\pi)^{1.5}|\Sigma|^5 X_1X_2X_3} Max\left\{X_1 - \frac{X_2X_1}{X_3}, 0\right\}$$

$$\bullet\ exp\left\{-\frac{1}{2}\underline{X}'\Sigma^{-1}\underline{X}\right\}dX_1dX_2dX_3$$

where Σ = variance and covariance matrix of $\ln\tilde{X}_1$, $\ln\tilde{X}_2$, and $\ln\tilde{X}_3$

$$\underline{X} = \begin{bmatrix} \ln X_1 - \hat{E}(\tilde{X}_1) + .5\sigma^2_{\ln\tilde{X}_1} \\ \ln X_2 - \hat{E}(\tilde{X}_2) + .5\sigma^2_{\ln\tilde{X}_2} \\ \ln X_3 - \hat{E}(\tilde{X}_3) + .5\sigma^2_{\ln\tilde{X}_3} \end{bmatrix}$$

$$\begin{aligned} \hat{E}(\tilde{X}_1) &= E(\tilde{L}_i)exp\{-\lambda Cov(\ln\tilde{L}_i, \ln\tilde{R}_m)\} \\ \hat{E}(\tilde{X}_2) &= (S_0 + \Pi_a)(1 + r_f) \\ \hat{E}(\tilde{X}_3) &= E(\tilde{L}_a)exp\{-\lambda Cov(\ln\tilde{L}_a, \ln\tilde{R}_m)\}\,. \end{aligned}$$

Please note that the meaning of equation (10) is exactly the same as that of equation (4).

We can determine an individual premium (Π_i) with equations (2), (9), and (10). As discussed in the *Joint Normality and CARA* case, however, we need another model for Π_a*. To develop a model for Π_a*, we consider the *payoff to the shareholders* and follow the procedure suggested by Stapleton and Subrahmanyam (1984) as an application of their model.[12]

For convenience, we synthesize two variables:

$\tilde{Y} = (S_0 + \Pi_a)(1 + \tilde{r}_A)$ and $\tilde{U} = \tilde{Y} - \tilde{L}_a + \Pi_a$.

Please note the problem associated with the distribution of $\tilde{U}$. Since $\tilde{U}$ is a linear combination of two random variables with lognormal distributions, $\tilde{U}$ is not lognormally distributed. Therefore, there might be a problem with applying the Stapleton and Subrahmanyam model (1984). This problem is not a serious one, however.[12]

We apply the Stapleton and Subrahmanyam model (1984), and the result is that

$$V_e = V_U N(d_1) - \frac{1}{1+r_f}\Pi_a N(d_2) \tag{11}$$

where V_U = value of $\tilde{U} = S_0 + \Pi_a - V_{L_a} + \frac{1}{1+r_f}\Pi_a$

V_{L_a} = value of $\tilde{L}_a = \frac{1}{1+r_f}E(\tilde{L}_a)exp\{-\lambda Cov(1n\tilde{L}_a, 1n\tilde{R}_m)\}$

$$d_1 = \frac{1n\left\{\frac{V_U}{\Pi_a}\right\} + 1nR_f + \frac{\sigma_U^2}{2}}{\sigma_U}$$

$$d_2 = d_1 - \sigma_U^{\%}$$

$$\sigma_U^2 = \sigma^2_{1n(1+\tilde{L}_a)} + \sigma^2_{1n\tilde{L}_a} - 2Cov(1n(1+\tilde{r}_A), 1n\tilde{L}_a).$$[13]

Since V_e (*the value of the payoff to shareholders*) should be equal to S_0 (*initial equity*) in equilibrium, Π_a* is an implicit solution of the following equation:

$$S_0 = V_U N(d_1) - \frac{1}{1+r_f}\Pi_a N(d_2). \tag{12}$$

Equations (2), (9), (10), and (12) constitute a pricing model for an insurance contract. The procedures approaching Π_i* are as follows:

- Procedure 1: Solve equation (12) for Π_a* using an iterative procedure.
- Procedure 2: Solve equation (10) with the Π_a* obtained by Procedure 1.
- Procedure 3: Solve equation (9).
- Procedure 4: Solve equation (2) with the results obtained by Procedures 2 and 3.

Sum of Individual Premiums

Now let's consider the insurance portfolio of an insurer. The payoff to the insurance portfolio is equal to the *aggregate claim* ($\tilde{L}_a$) from the insurance portfolio or the total asset of the insurer in the case of insolvency. In other words,

$$\begin{aligned} &Payoff\ \ to\ \ portfolio = Min\{\tilde{L}_a, (S_0 + \Pi_a)(1 + \tilde{r}_A)\} \\ &= \tilde{L}_a - Max\{\tilde{L}_a - (S_0 + \Pi_a)((1 + \tilde{r}_A), 0)\} \\ &= \sum_{i=1}^{N} \tilde{L}_i - \sum_{i=1}^{N} Max\left\{\tilde{L}_i - (S_0 + \Pi_a)(1 + \tilde{r}_A) \times \frac{\tilde{L}_i}{\tilde{L}_a}, 0\right\}, \end{aligned} \tag{13}$$

where N = the number of insurance contracts in the insurance portfolio.

Equation (13) states that policyholders receive aggregate claim payments if their company is solvent, or a residual value of the firm if their company is insolvent.

Since Π_a is the value of the payoff to the insurance portfolio,

$$\Pi_a = V\left[\sum_{i=1}^{N} \tilde{L}_i\right] - V\left[\sum_{i=1}^{N} Max\left\{\tilde{L}_i - (S_0 + \Pi_a)(1 + \tilde{r}_A) \times \frac{\tilde{L}_i}{\tilde{L}_a}, 0\right\}\right]. \tag{14}$$

Since the covariance in equations (3) and (9) and the integral in equation (4) and (10) are additive in $\tilde{L}_i$, it follows that

$$\Pi_a = \sum_{i=1}^{N} V[\tilde{L}_i] - \sum_{i=1}^{N} V\left[Max\left\{\tilde{L}_i - (S_0 + \Pi_a)(1 + \tilde{r}_A) \times \frac{\tilde{L}_i}{\tilde{L}_a}, 0\right\}\right] = \sum_{i=1}^{N} \Pi_i\ . \tag{15}$$

Obviously, the aggregate premium is the sum of all individual premiums.

MODEL ANALYSIS

Since our model consists of some complicated equations and provides an implicit solution, this study conducts a sensitivity analysis to test a validity of the model and to examine the effects of the variables on individual premiums.

Components of Insurance Premium

As a preliminary procedure, we identify the components of Π_i by scrutinizing equations (2), (3), and (9). The first term of equation (2), $V(L_i)$, represents the value of individual claims, and the second term represents the insolvency risk premium. As shown in equations (3) and (9), the value of individual claims (L_i) can be separated into two components: the value of individual claims without contingency of claims and the risk premium for the contingency of claims.

Therefore, Π_i includes three components:

$$\Pi_i = \text{component 1} - \text{component 2} - \text{component 3}, \tag{16}$$

where component 1 = value of individual claims without contingency,
component 2 = risk premium for the contingency of claims,
component 3 = insolvency risk premium.

Sensitivity Analysis

This study performs a sensitivity analysis with the initial values given in Table 1. The variables in the model are classified into three groups. The first group includes the variables beyond the scope of insurance, the second group includes the variables associated with insurers, and the third group includes the variables associated with individual insurance contracts. Table 2 shows the variables included in each group.

Among the variables beyond the scope of insurance we select r_f for a detailed examination. Table 3 shows a negative relationship between r_f and the two components of premium for both normal and lognormal cases. The effect of r_f on component 1 outweighs that of component 3. Consequently, an increase in r_f decreases individual premiums.[14] This result implies that an insurer can lower insurance premiums when higher investment income is expected as a result of higher r_f. In other words, the investment income of an insurer as a result of the time lag between premium receipt and claim payment is an important determinant of Π_i.

Among the variables associated with the insurer, we select S_0, $\sigma_{\tilde{Y}_A}$, and $\sigma_{\tilde{L}_a}$ for analysis. Table 4 shows a negative relationship between S_0 (initial equity) and component 3 (insolvency risk premium) for both normal and lognormal cases. As the equity of an insurer increases, the insolvency risk of the insurer decreases because of the equity cushion effect. As a result, the insolvency risk premium decreases, and the insurance premium increases.[15] This result implies that the equity of an insurer is an important determinant of Π_i.

The investment policy is another important factor that determines an insurer's insolvency risk. $\sigma_{\tilde{r}_A}$ (standard deviation of the rate of return of the investment portfolio) is chosen as a representative variable measuring the investment policy of an insurer. Table 5 shows a positive relationship between $\sigma_{\tilde{r}_A}$ and the component 3 of premium (insolvency risk premium) for both normal and lognormal cases. An increase in $\sigma_{\tilde{r}_A}$ means that an insurer takes a more aggressive or riskier investment policy. As a result, the insolvency risk premium increases, and the insurance premium should decrease.[16] This result implies that premiums of an insurer adopting an aggressive investment policy should be lower than those of another insurer with a conservative investment policy.

The effect of the underwriting policy on premiums is shown in Table 6. $\sigma_{\tilde{L}_a}$ (standard deviation of the aggregate claim) is chosen as a representative variable measuring the underwriting policy of an insurer. Table 6 shows a positive relationship between $\sigma_{\tilde{L}_a}$ and the component 3 of premium (insolvency risk premium) for both normal and lognormal cases. An increase of $\sigma_{\tilde{L}_a}$ means that an insurer takes a more lenient underwriting policy. As a result, the insolvency risk premium of the insurer should increase. Therefore, a negative relationship between $\sigma_{\tilde{L}_a}$ and insurance premium should exist, as shown in Table 6.[17] In other words, insurance premiums should be lower as the insolvency risk of an insurer is higher because of more lenient underwriting policy.

Since the insurance portfolio held by an insurer includes, generally, a large number of contracts to eliminate the contingency of claims except the systematic portion, we choose the correlation between individual claims and market rate of return to analyze the effect of the contingency of claims on Π_i. As shown in Table 7, $\rho_{\tilde{L}_i, \tilde{r}_m}$ and $\rho_{\ln\tilde{L}_i, \ln(1+\tilde{r}_m)}$ show a positive relationship with the component 2, meaning that the burden of an insurer to pay claims decreases as the correlation increases. Meanwhile, Table 7 shows that $\rho_{\tilde{L}_i, \tilde{r}_m}$ and $\rho_{\ln\tilde{L}_i, \ln(1+\tilde{r}_m)}$ have negative relationships with the component 3, meaning that the insolvency risk of an insurer decreases as the correlation increases. Table 7 shows that the positive effect on component 2 outweighs the negative effect on component 3. Consequently, an increase in $\rho_{\tilde{L}_i, \tilde{r}_m}$ and $\rho_{\ln\tilde{L}_i, \ln(1+\tilde{r}_m)}$ decreases the insurance premium.[18] In other words, an insurance premium should be lowered if the correlation between claims and market rate of return is high.

SUMMARY AND CONCLUSION

Most ratemaking principles or models for insurance pricing in the literature do not seem to incorporate at least one of the following three elements: contingency of claims, investment income of insurers, and insolvency risk of insurers. As far as we know, Doherty-Garven (1986) appears to be the only model that incorporates all three components and provides a model for the fair rate of return of an insurer or the aggregate premium of an insurer. The Doherty-Garven study has an important implication with regard to "excessiveness" and "adequacy" of the aggregate premium of an insurer. Although Phillips, Cummins, and Allen (1998) developed a dynamic model that disaggregates the aggregate premium by lines, they did not seem to address individual premiums.

We derived a model with which a premium could be assigned to an individual insurance contract. Our research is based on the Doherty-Garven (1986) study that is designed to derive a fair rate of return for property-liability insurance firms. But the focus of this paper is different because our model is designed to derive individual premiums rather than aggregate premiums. We think that our study has quite a meaningful implication in the real world, especially for determining fair individual insurance premiums. Since our model consists of several complicated equations and provides an implicit solution, this study performed a sensitivity analysis to verify the model and to scrutinize the effects of the variables on individual premiums. The sensitivity analysis showed that our model has validity and is consistent with the accepted knowledge.

The traditional method of insurance ratemaking is based on expected future claims and loading factors. This study suggests an alternative way of making and/or examining individual premiums by incorporating the contingency of claims, the investment income of an insurer, and the insolvency risk of an insurer. The solution of our study can be applied to validate insurance premiums not only for insurers but also for regulators. As indicated earlier in this paper, this study presents a model of key variables that ratemaking professionals should consider in the real world. According to our analysis, individual premiums based only on expected claims might be too high, and they should be adjusted downward in a competitive insurance market. Rate-makers should consider risk-free interest rates, initial equity, and the risks of investment and claims in addition to expected claims. Our simulation analysis presents how these variables should affect individual premiums. Furthermore, we believe that this study has implications for rate regulation in insurance. Regulators can apply our model to evaluate if individual insurance premiums are "excessive, inadequate, or unfair" by considering the nontraditional factors of ratemaking presented in the study.

For the insolvency risk of an insurer, it is also possible to incorporate the guaranty funds in our model, but it could not add much significance to this study. Obviously, one research project cannot address every challenging issue. We considered only net premium in this paper, and our model is a single-period model that employs a discrete time framework and a supply-side approach. The areas for further research that can be spawned by the work done here include:

- Extending the model to multi-period.
- Developing a model for gross premium.
- Employing a continuous time framework instead of a discrete time framework.
- Employing a demand-side approach instead of a supply-side approach.

Table 1. Initial Values for Numerical Illustrations

Variables	Initial Values
Initial Equity (S_0)	2000
Risk-Free Interest Rate (r_f)	0.08
Expected Market Rate of Return $[E(\tilde{r}_m)]$	$0.06 + r_f$
Standard Deviation of Aggregate Claims $(\sigma_{\tilde{L}_a})$	1500
Standard Deviation of Market Rate of Return (σ_m)	0.2
Correlation between L_a and $\tilde{r}_m$ $(\rho_{\tilde{L}_a, r_m})$	0
Correlation between $1nL_a$ and $1n(1+\tilde{r}_m)(\rho_{1nL_a, 1n(1+r_m)})$	0
Expected Aggregate Claims $[E(\tilde{L}_a)]$	10,000
Standard Deviation of Rate of Return of Investment Portfolio (σ_{r_A})	0.2
Expected Rate of Return of Investment Portfolio $[E(\tilde{r}_A)]$	$r_f + 0.06\frac{\sigma_{r_A}}{\sigma_m}$
Correlation between L_a and $\tilde{r}_A$ (ρ_{L_i, r_A})	ρ_{L_a, r_m}
Correlation between L_a and $1n(1+r_A)(\rho_{1nL_a, 1n(1+r_A)})$	$\rho_{1nL_a, 1n(1+r_m)}$
Expected Individual Claims $E(L_i)$	0.01
Standard Deviation of Individual Claims $(\sigma_{\tilde{L}_i})$	0.003
Correlation between L_i and $\tilde{r}_m$ $(\rho_{\tilde{L}_i, r_m})$	0
Correlation between $1nL_i$ and $1n(1+\tilde{r}_m)(\rho_{1nL_i, 1n(1+r_m)})$	0
Correlation between $\tilde{r}_A$ and $\tilde{L}_i$ (ρ_{r_A, L_i})	ρ_{L_i, r_m}
Correlation between $1n(1+\tilde{r}_A)$ and $1nL_i(\rho_{1n(1+r_A), 1nL_i})$	$\rho_{1nL_i, 1n(1+r_m)}$
Correlation between L_i and L_a $(\rho_{L_i, \tilde{L}_a})$	0
Correlation between $1nL_i$ and $1nL_a(\rho_{1nL_i, 1n\tilde{L}_a})$	0

Table 2. Classification of Variables

Group	Variables
Variables beyond the scope of insurance	$r_f, E(\tilde{r}_m), \sigma_m$
Variables associated with insurer	$S_0, \sigma_{\tilde{L}_a}, \rho_{\tilde{L}_a, r_m}, \rho_{\ln L_a, \ln(1+r_m)}, E(\tilde{L}_a), \sigma_{\tilde{r}_A}, E(\tilde{r}_A),$ $\rho_{L_a, r_A}, \rho_{\ln L_a, \ln(1+r_A)}$
Variables associated with insurance contract	$E(L_i), \sigma_{L_i}, \rho_{L_i, r_m}, \rho_{\ln L_i, \ln(1+r_m)}, \rho_{r_A, L_i},$ $\rho_{\ln(1+r_A), \ln L_i}, \rho_{L_i, L_a}, \rho_{\ln L_i, \ln L_a}$

Table 3. The Effect of Risk Free Interest Rate

r_f	Component 1	Component 2	Component 3	Π_i
		(Normal and CARA Case)		
0.06	0.0094339	0	0.0003461	0.0090878
0.07	0.0093458	0	0.0003323	0.0090135
0.08	0.0092592	0	0.0003190	0.0089402
0.09	0.0091743	0	0.0003063	0.0088680
0.10	0.0090909	0	0.0002942	0.0087967
		(Lognormal and CRRA Case)		
0.06	0.0094339	0	0.0002881	0.0091458
0.07	0.0093458	0	0.0002752	0.0090706
0.08	0.0092592	0	0.0002629	0.0089963
0.09	0.0091743	0	0.0002512	0.0089231
0.10	0.0090909	0	0.0002400	0.0088509

Component1: Value of individual claims without contingency.
Component 2: Risk premium for the contingency of claim.
Component 3: Insolvency risk premium.
Π_i = Component 1 – component 2 – component 3

Table 4. The Effect of Initial Equity

S_0	Component 1	Component 2	Component 3	Π_i
		(Normal and CARA Case)		
0000	0.0092592	0	0.0066463	0.0026219
1000	0.0092592	0	0.0007031	0.0085561
2000	0.0092592	0	0.0003190	0.0089402
3000	0.0092592	0	0.0001609	0.0090983
4000	0.0092592	0	0.0000850	0.0091742
		(Lognormal and CRRA Case)		
0000	0.0092592	0	0.0035081	0.0057511
1000	0.0092592	0	0.0006120	0.0086472
2000	0.0092592	0	0.0002629	0.0089963
3000	0.0092592	0	0.0001184	0.0091408
4000	0.0092592	0	0.0000540	0.0092052

Table 5. The Effect of Investment Policy

σ_{r_A}	Component 1	Component 2	Component 3	Π_i
		(Normal and CARA Case)		
0.16	0.0092592	0	0.0002078	0.0090514
0.18	0.0092592	0	0.0002601	0.0089991
0.20	0.0092592	0	0.0003190	0.0089402
0.22	0.0092592	0	0.0003841	0.0088751
0.24	0.0092592	0	0.0004546	0.0088046
		(Lognormal and CRRA Case)		
0.16	0.0092592	0	0.0001767	0.0090825
0.18	0.0092592	0	0.0002182	0.0090410
0.20	0.0092592	0	0.0002629	0.0089963
0.22	0.0092592	0	0.0003137	0.0089455
0.24	0.0092592	0	0.0003665	0.0088927

Component1: Value of individual claims without contingency.
Component 2: Risk premium for the contingency of claim.
Component 3: Insolvency risk premium.
Π_i = Component 1 – component 2 – component 3

Table 6. The Effect of Underwriting Policy

σ_{L_a}	Component 1	Component 2	Component 3	Π_i
		(Normal and CARA Case)		
500	0.0092592	0	0.0002206	0.0090386
1000	0.0092592	0	0.0002576	0.0090016
1500	0.0092592	0	0.0003190	0.0089402
2000	0.0092592	0	0.0004036	0.0088556
2500	0.0092592	0	0.0005083	0.0087509
		(Lognormal and CRRA Case)		
500	0.0092592	0	0.0001665	0.0090927
1000	0.0092592	0	0.0002067	0.0090525
1500	0.0092592	0	0.0002629	0.0089963
2000	0.0092592	0	0.0003317	0.0089275
2500	0.0092592	0	0.0003974	0.0088618

Table 7. The Effect of Contingency of Claims

ρ_{L_i, r_m} or $\rho_{\ln L_i, \ln(1+r_m)}$	Component 1	Component 2	Component 3	Π_i
		(Normal and CARA Case)		
–0.2	0.0092592	–0.0001667	0.0003503	0.0090756
–0.1	0.0092592	–0.0000833	0.0003347	0.0090078
0	0.0092592	0	0.0003190	0.0089402
0.1	0.0092592	0.0000833	0.0003032	0.0088727
0.2	0.0092592	0.0001666	0.0002872	0.0088054
		(Lognormal and CRRA Case)		
–0.2	0.0092592	–0.0001704	0.0003084	0.0091212
–0.1	0.0092592	–0.0000848	0.0002864	0.0090576
0	0.0092592	0	0.0002629	0.0089963
0.1	0.0092592	0.0000840	0.0002387	0.0089365
0.2	0.0092592	0.0001673	0.0002145	0.0088774

Component1: Value of individual claims without contingency.
Component 2: Risk premium for the contingency of claim.
Component 3: Insolvency risk premium.
Π_i = Component 1 – component 2 – component 3

NOTES

[1] For the clear concepts of excessiveness, adequacy, and fairness of insurance premiums, see Freifelder (1976).

[2] "Assessments levied by state guaranty funds to pay the obligations of insolvent insurers totaled $2.5 billion from 1969 to 1988. Obviously, insurance debt carries default risk that must be taken into account in pricing" (Cummins, 1992, pp. 144–145).

[3] The ratemaking principles mentioned here are mainly based on three books: Freifelder (1976), Gerber (1979), and Goovaerts, De Vylder, Haezendonck (1984).

[3-1] The financial models discussed in the following several paragraphs are mainly based on three papers: Cummins (1991), D'Arcy and Doherty (1988), D'Arcy and Garven (1990).

[4] The Insurance CAPMs are the insurance pricing models based on the CAPM.

[5] For the clear concepts of the aggregate wealth and the representative investor, see Brennan (1979) and Stapleton and Subrahmanyam (1984).

[6] See Stapleton and Subrahmanyam (1984) p. 214, equation (16).

[7] This approach is the same as that of Doherty and Garven (1986).

[8] See Brennan (1979) p. 64, equation (39).

[9] Equation (7) is the same as that of Doherty and Garven (1986) p. 1038, equation (19), except for the fund generating coefficient and corporate taxes.

[10] See Stapleton and Subrahmanyam (1984) p. 216, equation (25).

[11] See Stapleton and Subrahmanyam (1984) pp. 223–224.

[12] "Strictly speaking, if $\tilde{Y}$ and $\tilde{L}_a$ are lognormal, U can not be lognormal. However, if $\tilde{Y}$ and $\tilde{L}_a$ follow a joint Wiener process in continuous time, then $\tilde{Y}$, $\tilde{L}_a$, and U will be joint lognormal in discrete time if continuous hedging is possible in the Black-Scholes-Merton sense. The analysis here may be regarded as an approximation or the risk neutral valuation of end-of-period payoffs generated by continuous joint Wiener processes with continuous hedging using the Cox-Ross (1976) argument. However, if the approximation is regarded as inappropriate, the integration can be carried out over the joint distribution of $\tilde{Y}$ and $\tilde{L}_a$ by numerical methods." (Stapleton and Subrahmanyam, 1984, p. 223, with some changes in notation).

[13] Equation (11) is the same as that of Doherty and Garven (1986) p. 1041, equation (30) except for the fund generating coefficient and the corporate taxes.

[14] See equation (16).

[15] See equation (16).

[16] See equation (16).

[17] See equation (16).

[18] See equation (16).

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