Pricing of a European Call Option on Pension Annuity Insurance*

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Abstract: We present a European call option that is defined on a pension annuity insurance contract. This option gives the holder of the contract the opportunity to buy a pension annuity benefit for a given (strike) price at the age of retirement or any other age. Thus instead of contributed monthly payments to the pension fund, the holder of the option contract would be entitled to buy this European call option as insurance and to fix the terms of payment in advance. Consequently, holders could invest their money in the market at their own discretion. This approach to pension insurance introduces traditional pension pricing to the financial world. We present the model and illustrate the pricing for some particular cases, and we draw comparisons to traditional pension contracts. In doing so, we use methods from actuarial mathematics and mathematics of finance. [Key words: Binomial model, European call option, Life insurance, Pension insurance]

INTRODUCTION

In traditional pension insurance plans, policyholders pay annual premiums to pension funds and/or insurance companies through the age of retirement and, in exchange, receive an annual income (pension benefit) from the time of retirement through death. In this paper we propose a concept regarding options defined on pension annuity insurance, whereby insured parties aged x can buy European call options on their pension annuity benefit. This call option allows policyholders to buy their discounted annual pension annuity benefit from the options writers, at a

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defined strike price, prior to or at the age of retirement. Thus instead of contributing monthly payments to the pension fund, investors would be entitled to buy this European call option as an insurance contract. In the meantime they could invest their money in the market at their own discretion. This approach to pension insurance introduces traditional pension insurance to the financial world.

In the practical world, most U.S.-based insurance companies offer holders tax-sheltered saving plans, also known as variable annuity contracts—the long-term option for annuitizing their policies at a predetermined rate over a prespecified period of time. If annuity rates at the time of annuitization are more favorable than the contractually specified value, policyholders may demand current rates or go elsewhere. Conceptually, this is a call option on annuity, whose purchase factors can be viewed as the right, but not the obligation, to purchase a fixed immediate life annuity for a predetermined strike price during the life of the contract (see Milevsky and Promislow (2001)).

One of the studies that combines call options with pension plans is Joseph and Kam (1995), which extends the "pension put" option—the theoretical approach of Sharpe (1976) and Bicksler and Chen (1985)—to a "pension call" model, in order to describe the general phenomenon of unwillingness to terminate overfunded plans by fund sponsors. A pension put, as described by Sharpe (1976), is the value of the sponsor's right to abandon an underfunded pension plan. If the sponsor exercises the pension put option (i.e., abandons the underfunded pension plan), it leaves the responsibility of the shortfall to either the beneficiaries or the PBGC (Pension Benefit Guarantee Corporation) in the short term. Following the general argument in Sharpe (1976), to preserve the value of the pension put, the sponsor would not be motivated to terminate an underfunded plan. Consequently, the value to the sponsor of early termination of a defined benefit plan is similar to the exercise value of a call option on the pension asset portfolio. The exercise price of the call option is equal to the vested benefit at the time plan termination is considered. Another interesting study is that conducted by Raman and Gautam (1997), which uses a continuous time model to determine an intervention policy for the PBGC. They use option pricing techniques to determine the time of an active intervention. This intervention is necessary because of the deficits with which the defined benefit pension plans must deal. The earliest studies dealing with the problem of the difference of pension plan assets and retirement benefits (obligations) are Williger (1985) and Bodie (1991). Another type of pension plan in countries such as Australia, Canada, and the United States promised beneficiaries the greater of the values of a defined benefit and a defined contribution ("Greater of Benefits," GOB).

This typical contingent claim, the GOB, can be regarded as a call option with a zero exercise price on the maximum of two risky assets: the defined benefit value and the defined contribution. The GOB was studied by Britt (1991), Bell and Sherris (1991), Sherris (1993, 1995), Cohen and Bilodeau (1996), and Bancinello (2000). In a recent study, Milevsky and Promislow (2001) attempt to value options on mortality-contingent claims by stochastically modeling the future hazard rate and interest rate. The paper focuses on European-style options on the mortality-contingent claim that pays one lump sum upon survival of a prespecified period. This is also known as an endowment policy, in contrast to traditional life annuity, and is quite similar to a zero-coupon bond. Finally, the latest research done by Ostaszewski (2002) proposes that all products that provide protection again random losses such as life and disability insurance, as well as annuities, can be viewed as financial instruments. For instance, in the case of term life insurance, Ostaszewski proposes looking at this insurance contract as a put option on human capital, with an exercise price equal to the amount of insurance, where the strike price is zero.

This brings us to our research, in which we suggest looking at traditional pension insurance, which will be discussed in the following section, as a European call option, where the underlying asset is the pension annuity of the option holders. The exercise date of the option is the day of retirement of the option holder or several years before the day of retirement (as explained below). Option holders would be entitled to exercise the option only if they survive until the exercise date. Another interesting case would be when this option is tradable, such that dealers would be able to buy this call option from their holders and decide whether or not to exercise the option on the exercise date. If the first option holders survive until their age of retirement (by first option holders we refer to the insured parties for whom the call option is on their pension annuity). In this case, the dealers who buy this option will receive the pension benefit instead of the first holder of the option. Thus, as mentioned previously, and also as suggested by Milevsky and Promislow (2001) and Ostaszewski (2002), this approach to pension insurance introduces traditional pension pricing to the financial world, which is widely researched in many of the current studies in the actuarial science and finance fields.

The structure of this proposal is as follows: the second relates to two types of traditional pension insurance, of which first deals with contracts according to which the insured parties pay the pension fund or insurance company a single premium at the age of retirement and receive monthly pension benefits from the age of retirement through death. The second presents the most common pension insurance contracts according to which the insured parties pay monthly premiums and receive pension benefits at the age of retirement, which are paid monthly until their death. The third section presents a European call option on pension insurance, in which the insured parties pay one premium at age x and can buy their discounted pension benefits at the strike price on the exercise date. We use the binomial model as the valuation model and provide calculations for the price of the option in cases where the exercise date is at the age of retirement and when the exercise date falls prior to the age of retirement. The fourth section concludes the paper.

For the sake of convenience, and without loss of generality, we assume for all pension contracts that the insured parties receive their pension annuity only as a result of retirement due to age. Therefore, if they withdraw from the pension fund for other reasons, such as illness, they will receive no annuity. We clearly ignore expenses, profits, and other administrative charges and thus assume that everything is presented on a net basis.

TRADITIONAL PENSION INSURANCE CONTRACTS

Consider a case in which an insured party, age x, buys pension insurance for a single premium, guaranteeing an annuity paid monthly from the age of retirement through death—a traditional pension insurance for age-related retirement. Denote by B the monthly benefit of the pension annuity and by $({}_nE_x)$ the present value of a pure endowment of \$1 payable at age $x + n$, for an insured party now aged x, if said party is still alive in the next n years. Therefore, $({}_nE_x)$ can be written by:

$$({}_nE_x) = ({}_np_x)\left(\frac{1}{1+r}\right)^n, \tag{1}$$

where r is the yearly risk-free interest rate, and $({}_np_x)$ is the probability of the insured, aged x, to continue living for the next n years—a well known fact based on actuarial life tables. Now, denote by ${}_nS_x$ the single premium that is paid by an insured age x, n years from retirement, for monthly benefits, B (i.e., the yearly benefits are $12*B$), which will be received from the age $x + n$ and through death. $({}_nS_x)$ is therefore given by:

$${}_nS_x = B({}_nE_x)\ddot{a}^{(12)}_{x+n}, \tag{2}$$

where $\ddot{a}^{(12)}_{x+n}$ is the present value of a whole life annuity due of \$1 per year, payable monthly by an insured party aged $x + n$ (i.e., $\frac{1}{12}$ per month).

Note that if the insured party buys this contract for a premium that is paid monthly, we can denote this premium by

$$ {}_nS_x^{(12)} = \frac{B({}_nE_x)\ddot{a}_{x+n}^{(12)}}{\ddot{a}_{x:n}^{(12)}}. \tag{3} $$

Numerical Example: Traditional Pension Insurance

Consider policyholders whose ages are as specified in the table below and who buy pension insurance contracts for a single premium. Assume the annuity benefit is B = \$2000, paid monthly. Furthermore, assume that the risk-free interest rate per annum (yearly basis) is provided in the table below and that the age of retirement is $x + n = 65$. The actuarial life table that we choose to work with is *A*(67–70)–*male*, which is based on experience within these years of insured parties of insurance companies in the United Kingdom (this table was used by Israeli insurance companies until about ten years ago). Then from (2) we obtain the single premiums:

Table 1: Single premium at age $x - {}_nS_x$

Age x	r = 10%	r = 5%	r = 3%	r = 0%
30	\$5,082	\$34,455	\$77,140	\$282,593
40	\$13,552	\$55,128	\$103,740	\$286,082
50	\$33,880	\$91,880	\$143,640	\$293,059
65	\$169,402	\$229,709	\$266,000	\$342,880

and from (3), we derive the monthly premiums given in Table 2:

Table 2: The monthly premium ${}_nS_x^{(12)}$

Age x	r = 10%	r = 5%	r = 3%	r = 0%
30	\$42.3	\$175.5	\$304.7	\$705.8
40	\$121.4	\$330.5	\$511.9	\$1,009.6
50	\$368.2	\$757.5	\$1,044.5	\$1,734.1

EUROPEAN CALL OPTION ON PENSION INSURANCE

Consider a European call option, which entitles holders to buy the underlying asset by a certain date and at a fixed price. The underlying asset in our case is the pension annuity of the option holders. Consider the case in which the exercise date is the age of retirement of the contract holders.

Note that there are many reasons why insured parties would prefer to buy this call option, as opposed to the regular insurance contract previously mentioned. (1) The insured parties believe they can invest their money better than the pension fund/insurance company. (2) The insured parties do not currently have the financial ability to buy the pension insurance, but they may be in a position to do so in the future. (3) The insured parties can enjoy their money while they are still young and do not need to pay a monthly premium to the pension fund/insurance company for many years. They do, however, have the opportunity to buy the life annuity in the future at a reasonable price.

Strike Price at Age of Retirement

Consider an insured party aged x who buys a call option on a discounted pension annuity. Assume the monthly benefit pension annuity is B, and the age of retirement is $x + n$. This means that the price of underlying security from the expiration is

$$ {}_nS_x = B\ddot{a}^{(12)}_{x+n}({}_nE_x) . \tag{4} $$

Note that (4) is exactly the same as presented in (2). To evaluate this call option, we assume that ${}_nS_x$, which we call "annuity function" (a.f.), follows a multiplicative binomial process over discrete periods. By a.f. we refer to the function that describes, in each time unit, the value of the insurance contract in case of survival/death of the insured. If the insured party dies within the next time unit, a.f. is 0, and if the insured party survives within the next time unit, a.f. becomes ${}_{n-1}S_{x+1}$. Note that the underlying stochastic variable here is the mortality rates of the insured parties aged x, while the function a.f. is analog to the stock price process, which follows the binomial model in the traditional discrete pricing of European call options. For the sake of convenience, consider the mortality rates on a yearly basis—the actuarial life table defined in years.

Now the rate of return on the a.f. over each year can have two possible values:

(i) u_i is defined as (1+ (percentage of upwards change of the a.f.)) on year $[i-1, i]$ for $i = 1, 2, ..., n$, with probability $({}_1p_{x+i-1})$, which is the probability of the insured party, aged $x + i - 1$, to survive the next year.

(ii) d_i is defined as (1– (percentage of downwards change of the a.f.)) with probability $({}_1q_{x+i-1}) = 1 - ({}_1p_{x+i-1})$.

We start with the evaluation of u_1 and d_1. For this, we need to find the value of the pension benefit annuity at the beginning of the insurance contract—at time 0—and at the end of the year at time 1. Thus u_1 and d_1 can be written by

$$u_1 = \frac{({}_{n-1}S_{x+1})}{({}_nS_x)} = \frac{B\ddot{a}^{(12)}_{x+n}({}_{n-1}E_{x+1})}{B\ddot{a}^{(12)}_{x+n}({}_nE_x)} = \frac{{}_{n-1}P_{x+1}}{({}_np_x)v} \tag{5}$$

$$d_1 = 0 .$$

Without loss of generality, we assume that the insured parties are able to exercise the option only if they continue to live over the next n years—until the exercise date. We also assume there are no dividend payments in this model, that the interest rate is constant and positive, and that there are no taxes, transaction costs, or margin requirements.

Let r denote the risk-free interest rate over each year plus 1. We require $u_i > r > d_i$ for $i = 1, 2, ..., n$. Let C be the value of this call option at time 0, and C_{u_i} be its value at the end of year $[i-1, i]$ for $i = 1, 2, ..., n$ if the insured survives this year. And C_{d_i} is the value of the option at the end of this period if the insured dies within the next year. Note that in our case if the insured party does not survive out the year $[i-1, i]$ for $i = 1, 2, ..., n$, then $C_{d_i} = 0$.

To illustrate this, consider a call option of $n = 1$ year. We know that in the terms of the contract and a rational exercise policy we infer that $C_{u_1} = max[0, u_1({}_1S_x) - K]$, where K is the strike price. Therefore

C →

$C_{u_1} = \max[0, u_1({}_1S_x) - K]$

with probability p_x.

C →

$C_{d_1} = 0$

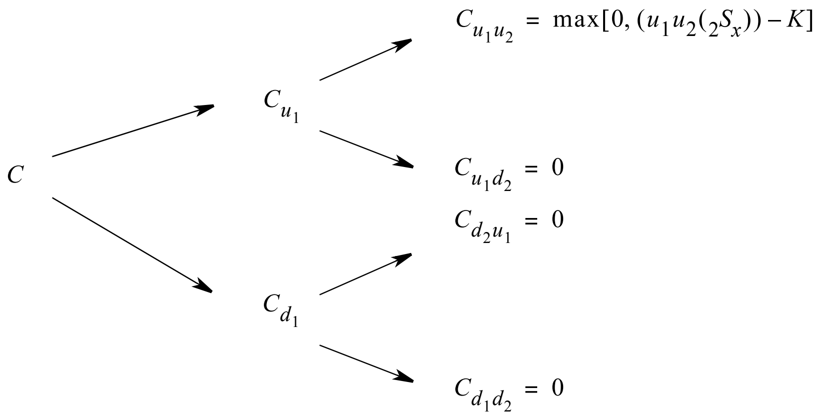
with probability q_x.

Consequently, if there are no riskless arbitrage opportunities, according to the same ideas of the classical binomial model (see, for example, Cox and Rubinstein, 1985, ch. 5), we get:

$$C = \frac{C_{u_1}}{u_1} = {}_1S_x - \frac{K}{u_1} \text{ where } \left(u_1 > \frac{K}{{}_1S_x}\right). \quad (6)$$

Note that the h ratio from the binomial model in this case is: $h = 1 - \frac{K}{{}_1S_x}$. In the event of a strategy without arbitrage opportunities in the aforementioned types of insurance, we buy h pension insurance contracts and sell in short one call option on the pension insurance.

If we consider a call option for $n = 2$ time periods, then similarly we obtain:



and the value of the call option is

$$C = ({}_2S_x)C - \frac{K}{\pi_{i=1}^{2} u_i} \text{ where } \pi_{i=1}^{2} u_i > \frac{K}{({}_2S_x)} \quad (6)$$

when

$$u_1 = \frac{({}_1S_{x+1})}{({}_2S_x)} = \frac{B\ddot{a}_{x+2}^{(12)}({}_1E_{x+1})}{B\ddot{a}_{x+2}^{(12)}({}_2E_x)} = \frac{({}_1p_{x+1})}{({}_2p_x)v}$$

$$u_2 = \frac{({}_0S_{x+2})}{({}_1S_{x+1})} = \frac{B\ddot{a}^{(12)}_{x+2}({}_0E_{x+2})}{B\ddot{a}^{(12)}_{x+2}({}_1E_{x+1})} = \frac{1}{({}_1p_{x+1})v}.$$

Consider now a call option for n years. The current value of this call option is:

$$C = ({}_nS_x) - \frac{K}{\pi^n_{i=1}u_i} \quad \forall \quad \pi^n_{i=1}u_i > \frac{K}{({}_nS_x)}, \tag{8}$$

where

$$u_i = \frac{({}_{n-1}p_{x+i})}{({}_{n-i+1}p_{x+i-1})v} \quad \forall \quad i = 1, \ldots, n-1 \tag{9}$$

$$u_n = \frac{1}{{}_1p_{x+n-1}v}.$$

It follows from (9) that

$$\pi^n_{i=1}u_i = \frac{1}{({}_np_x)v^n}. \tag{10}$$

Thus from (4), (8), and (10) we get the current value of this call option:

$$C = ({}_nS_x) - K({}_nE_x) = [B\ddot{a}^{(12)}_{x+n} - K]({}_nE_x) \quad \forall \quad B\ddot{a}^{(12)}_{x+n} > K. \tag{11}$$

Note that in this call option, if the insured party dies before the exercise date of the option, said party loses the price of the option C paid at age x. However, the insurance company may, in this case, decide to pay the insured party's family a percentage of C to encourage them to buy this option. Acknowledge that we can represent C by

$$C = [B\ddot{a}^{(12)}_{x+n}({}_np_xv^n) - Kv^n] + [1 - {}_np_x]Kv^n \quad \forall \quad B\ddot{a}^{(12)}_{x+n} > K, \tag{12}$$

which is easier to understand. $[B\ddot{a}^{(12)}_{x+n} - Kv^n]$ is the financial value of the option. If $K = 0$, the price of the call option is exactly ${}_nS_x$ – the single premium from (2), which is h paid n years from the age of retirement. Furthermore, $[1 - {}_np_x]Kv^n$ is the extra premium, which the option holders do not have to pay if they do not survive until the exercise date of the option.

Note that this call option can be tradable in such a manner that will enable dealers to buy it from the holders and decide whether or not to exercise the option on the exercise date. If the first option holders survive until their age of retirement (by first option holders we refer to the insured parties for whom the call option is in their pension annuity). In this case, the dealers who buy this option will receive the pension benefit instead of the first holder of the option.

Note that the result in (11) means that the call option for n periods could be considered as a call option for one period of time, when the length of this period is considered to be n years (see the Appendix).

Numerical Example: Strike Price at Age of Retirement

Consider insured parties whose ages are as specified in the table below and who are interested in buying a European call option on their pension annuity of $2000 per month, which will be paid from the age of retirement through death. Assume that the age of retirement is 65. The risk-free interest rate per annum (yearly) and the strike prices are provided in Table 3. The actuarial life table is A(67–70)–*male*. From (11) we obtain the prices of the European call option given in the table below. Note that in all the numerical examples we have added another column, which presents financial present value, $C + K(v^{65-x})$. This illustrates the difference between the actuarial and the financial present value, which the insured parties will invest in buying these call options. We can see in the present value column that always $[C + K(v^{65-x}) > ({}_{65-x}S_x)]$.

Table 3. European call option for particular cases of strike prices and interest rates – strike price at age of retirement

$r = 10\%$

Age x	K	${}_{65-x}S_x$	C	Present value $C + K(v^{65-x})$
	$4,500	$5,082	$4,947	$5,107
30	$10,000	$5,082	$4,782	$5,137
	$30,000	$5,082	$4,182	$5,250
	$20,000	$13,552	$11,952	$13,798
40	$30,000	$13,552	$11,152	$13,921
	$50,000	$13,552	$9,552	$14,167
	$30,000	$33,880	$27,880	$35,062
50	$40,000	$33,880	$25,880	$35,456
	$50,000	$33,880	$23,880	$35,850

$r = 5\%$

Age x	K	${}_{65-x}S_x$	C	Present value $C + K(v^{65-x})$
	$30,000	$34,445	$29,995	$35,434
30	$40,000	$34,445	$28,455	$35,707
	$50,000	$34,445	$26,955	$36,020
	$50,000	$55,128	$43,128	$57,893
40	$60,000	$55,128	$40,728	$58,446
	$70,000	$55,128	$38,328	$58,999
	$60,000	$91,880	$69,577	$98,438
50	$80,000	$91,880	$61,377	$99,858
	$120,000	$91,880	$44,977	$102,699

$r = 3\%$

Age x	K	${}_{65-x}S_x$	C	Present value $C + K(v^{65-x})$
	$70,000	$77,140	$56,840	$81,717
30	$90,000	$77,140	$51,040	$83,025
	$110,000	$77,140	$45,240	$84,332
	$90,000	$103,740	$68,640	$111,625
40	$100,000	$103,740	$64,740	$112,501
	$120,000	$103,740	$56,940	$114,253
	$120,000	$143,640	$78,840	$155,863
50	$140,000	$143,640	$68,040	$157,901
	$160,000	$143,640	$57,240	$159,938

We see, for example, that for insured parties age $x = 30$ who buy a call option for $n = 35$ years at a risk-free interest rate of 5%, the call option price is C = $29,995, instead of paying ${}_{35}S_{30}$ = $34,455 at age 30. If the age of the insured party is 30, he can invest the extra money (${}_{35}S_{30} - C$ = $4,460) at a 5% interest rate per year, and the insured party will have $24,601 at the age 65, while the strike price at age 65 is $30,000. This extra "forfeiture" of $5,399 can therefore be understood as the right to choose whether to exercise the call option. However, if the insured party invests this extra $4,460 over the 35 years, for example, and receives 6% per year in the free market, after 35 years that same party will have $34,280, meaning a profit of $4,280.

Therefore, the outcome depends on the investor's horizon. If the investors think that they can invest their money better than the insurance company or pension fund, then it is more worthwhile for them to purchase this call option. Furthermore, it allows them the choice of exercising the option or not, in the event they are not financially well off at the age of retirement or if they believe they do not have many more years to live. In this case they can save their money, instead of paying it to the insurance company or pension fund for these to profit. In Table 4 we can see the strike prices for low prices of the call option contract.

Table 4. Strike prices for low prices of the call option C

$r = 10\%$

Age x	K	${}_{65-x}S_x$	C	Present value $C + K(v^{65-x})$
	\$165,935	\$5,082	\$100	\$6,005
30	\$152,074	\$5,082	\$500	\$5,911
	\$134,747	\$5,082	\$1,000	\$5,795
	\$168,076	\$13,552	\$100	\$15,613
40	\$162,779	\$13,552	\$500	\$15,524
	\$156,157	\$13,552	\$1,000	\$15,413
	\$168,903	\$33,880	\$100	\$40,534
50	\$166,914	\$33,880	\$500	\$40,458
	\$164,427	\$33,880	\$1,000	\$40,363

$r = 5\%$

Age x	K	${}_{65-x}S_x$	C	Present value $C + K(v^{65-x})$
	\$229,020	\$34,445	\$100	\$41,619
30	\$226,299	\$34,445	\$500	\$41,526
	\$222,898	\$34,445	\$1,000	\$41,409
	\$229,786	\$55,128	\$100	\$67,956
40	\$227,630	\$55,128	\$500	\$67,720
	\$225,561	\$55,128	\$1,000	\$67,609
	\$229,453	\$91,880	\$100	\$110,471
50	\$228,463	\$91,880	\$500	\$110,395
	\$227,225	\$91,880	\$1,000	\$110,299

$r = 3\%$

Age x	K	${}_{65-x}S_x$	C	Present value $C + K(v^{65-x})$
	\$265,653	\$77,140	\$100	\$94,509
30	\$264,265	\$77,140	\$500	\$94,415
	\$262,530	\$77,140	\$1,000	\$94,299
	\$265,744	\$103,740	\$100	\$127,021
40	\$264,720	\$103,740	\$500	\$126,932
	\$263,441	\$103,740	\$1,000	\$126,821
	\$265,815	\$143,640	\$100	\$170,717
50	\$265,073	\$143,640	\$500	\$170,640
	\$264,145	\$143,640	\$1,000	\$170,545

Some might argue that this could be an issue pertaining to adverse selection, that is to say, only healthy people would exercise their options. This arguement can be alleged by considering that pension funds/insurance companies evaluate the contracts with the actuarial life table of healthy people, thus taking the issue of *adverse selection* into consideration in the pricing of the contracts. It also can be resolved by selling European call options on an exercise date, which is several years earlier than the age of retirement. In such a case the holders of the options do not know what their health situation will be when they reach retirement age.

Exercise Date before the Age of Retirement

Assume that insured parties age x purchase European call options as presented above, where the exercise date is $x + m$ and the age of retirement is $x + m + n$. Using the same method as previously presented, we obtain the price of the call option C to be:

$$C = ({}_{m+n}S_x) - K({}_mE_x) = [B\ddot{a}^{(12)}_{x+n+m}({}_{n+m}E_x) - K({}_mE_x)] \quad \forall \quad B\ddot{a}^{(12)}_{x+m+n}[({}_np_{x+m})v^n] > K. \tag{13}$$

Numerical Example: European Call Option—Exercise Date before the Age of Retirement

Consider, for example, insured parties whose ages are as specified in Table 5 who want to purchase this European call option on their pension annuity of \$2,000 a month. Suppose the age of retirement is $x + m + n = 65$, and the exercise date is at age $x + m = 55$. The risk-free interest rate per year and strike prices are provided in the table below. The actuarial life table is *A*(67–70)–*male*.

From (13), we obtain the prices of the European call option given in Table 5.

Table 5. European call options for strike prices on exercise date before age of retirement

$r = 10\%$

Age x	K	${}_{65-x}S_x$	C	Present value $C + Kv^m$
	\$4,500	\$5,082	\$4,693	\$5,108
30	\$10,000	\$5,082	\$4,218	\$5,141
	\$30,000	\$5,082	\$2,489	\$5,258
	\$20,000	\$13,552	\$9,029	\$13,817
40	\$30,000	\$13,552	\$6,768	\$13,950
	\$50,000	\$13,552	\$2,245	\$14,215
	\$30,000	\$33,880	\$15,814	\$34,442
50	\$40,000	\$33,880	\$9,792	\$34,629
	\$50,000	\$33,880	\$3,770	\$34,816

$r = 5\%$

Age x	K	${}_{65-x}S_x$	C	Present value $C + Kv^m$
	\$30,000	\$34,445	\$26,160	\$35,019
30	\$40,000	\$34,445	\$23,395	\$35,207
	\$50,000	\$34,445	\$20,630	\$35,395
	\$50,000	\$55,128	\$32,409	\$56,460
40	\$60,000	\$55,128	\$27,865	\$56,726
	\$70,000	\$55,128	\$23,321	\$56,992
	\$60,000	\$91,880	\$46,285	\$93,297
50	\$80,000	\$91,880	\$31,087	\$93,769
	\$120,000	\$91,880	\$691	\$94,714

$r = 3\%$

Age x	K	${}_{65-x}S_x$	C	Present value $C + Kv^m$
	\$70,000	\$77,140	\$45,836	\$79,268
30	\$90,000	\$77,140	\$36,892	\$79,877
	\$110,000	\$77,140	\$27,948	\$80,485
	\$90,000	\$103,740	\$49,171	\$106,939
40	\$100,000	\$103,740	\$43,108	\$107,294
	\$120,000	\$103,740	\$30,981	\$108,004
	\$120,000	\$143,640	\$43,247	\$146,760
50	\$140,000	\$143,640	\$26,515	\$147,280
	\$160,000	\$143,640	\$9,782	\$147,799

In this type of call option, the pension fund/insurance company may grant option holders the opportunity to decide what to do with their pension from the exercise date through time of retirement. One of the possibilities option holders could be given would be to grant the pension fund/insurance company the opportunity to invest the pension funds of the insured parties in the free market by buying bonds or investing in stocks. Note that we can extend this kind of option to payments made to insured parties until their deaths or to their spouses if one or the other does not survive within the next $m + n$ years, for the remainder of their lifetimes.

CONCLUSIONS

We introduce a concept regarding a European call option defined on pension annuity insurance, whereby insured parties can buy this option on their pension annuity benefit, granting them the opportunity to buy their discounted annual pension annuity benefit prior to or at the age of retirement, from the options writers, at a defined strike price. The use of this European call option for pensions is a new method that enables individuals to subscribe to a pension annuity at a later age, fixing the terms of payment in advance, while the current value paid by the individual is somewhat higher than standard pension annuity insurance. However, the European call option contract can only be advantageous for individuals who purchase a pension annuity contract if they are still alive at a specific date (later).

We believe that this new contract may lead more individuals to purchase pension insurance when they are young, by purchasing a European call option on pension annuity rather than a full pension insurance. The use of the option can be relevant in an imperfect capital market where young people cannot borrow money to pay for a pension on the basis of higher income or wealth in the future. Alternatively, individuals may want to invest the annual premiums for insurance on their own and pay a lump sum payment (exercise price) at a future date to exercise the option.

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APPENDIX

Consider a European call option for an insured party, age x, on a pension annuity. The monthly pension benefit is B, which is paid from the age of retirement through death. We assume a yearly interest rate, which is a deteministic one. The age of retirement is $x + n$. Consider this call option for a one-time period, which is defined to be n years. The h ratio in our case is

$$h = \frac{C_u}{{}_nS_x u} \tag{A.1}$$

Denote by $PORT$ the portfolio value at the beginning of the contract, and by $PORT_u$ and $PORT_d$ the value of the portfolio at the end of this one-time period (which is equal to n years) if the underlying asset goes upward downward, respectively. It is clear that in the event the insured party dies during the next n years, then $PORT_d = 0$. If the insured party lives during the next n years, the value of the portfolio at the end of n years is

$$PORT_u = h({}_0S_{x+n}) - C_u. \tag{A.2}$$

In case of no arbitrage opportunities

$$PORT = h({}_nS_x) - C = PORT_u({}_nE_x) = PORT_d({}_nE_x)\,. \tag{A.3}$$

Now under $C_u = \max[0, u({}_nS_x) - k]$ and $u = \frac{({}_0S_{x+n})}{({}_nS_x)}$, we obtain the value of the call option

$$C_u = ({}_nS_x) - K({}_nE_x) \ \forall \quad B\ddot{a}^{(12)}_{x+n} > K \tag{A.4}$$

which is exactly as presented in (11).